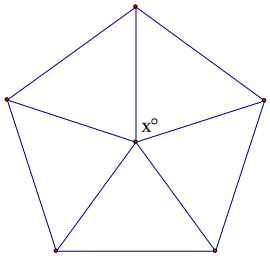
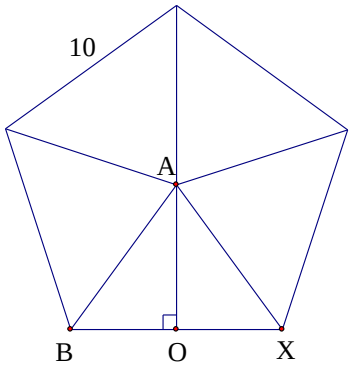
11.3 Area of a Regular Polygon - Notes

This is a regular pentagon. What is the measure of the angle at x?

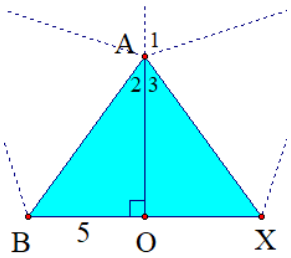


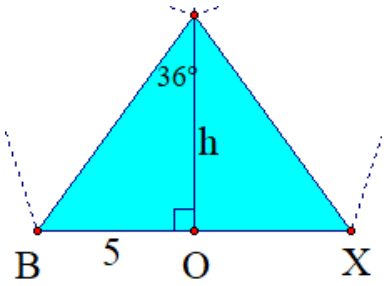
If you divided 360^0 by 5 then you made the right move. Each of the spokes such as $\overline{AB}$ and $\overline{AX}$ is called a radius of the regular polygon because each would be a radius for the circle circumscribed around the polygon. They are all congruent. That makes ΔBAX an isosceles triangle.

$\overline{AO}$ is drawn to the midpoint of $\overline{BX}$. For an isosceles triangle the median is also an altitude and angle bisector so we get congruent angles at the vertex and right angles at the base. (By the way, $\overline{AO}$ is called an apothem)



Since you figured that $m\angle 1 = 72$ then what do you get for $m\angle 2$?
Also, hopefully you see where the length BO = 5 came from…





The next thing to do would be a little trigonometry.

$$\tan 36° = \frac{5}{h}$$

$$h = \frac{5}{\tan 36°}$$

$$h \approx 6.88$$

So, the area of the blue triangle could be found with: $\text{area} = \frac{1}{2} b \cdot h$

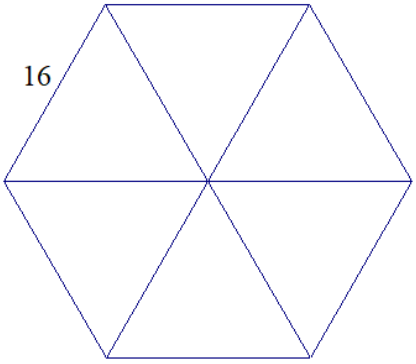
$$\text{area} = \frac{1}{2}(10)\cdot(6.8819096)$$

$$\text{area} \approx 34.40954801$$

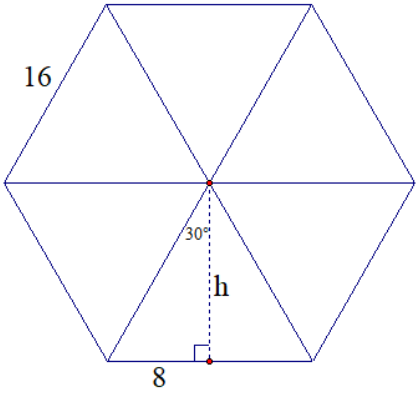
However, this isn't the entire pentagon. How would you get that?

If you said there are five triangles, then BAM! Multiply the area of the one blue triangle by 5.
So, the pentagon area is about 172.05 square units.

ex 1 Try the same process to find the area for this regular hexagon.



The angle in the middle is 360 divided by 6 which is 60 degrees. Dropping an altitude to the midpoint of a side gives a 30 degree angle and part of the base of that triangle is 8. Using tangent we can find the height to be about 13.8564. So the area of that bottom triangle is about ½ (16)(13.8564) or 110.85. The last step is to remember there are six of these triangles. So, our total hexagon area is about 665.11 square units.



Remember, the area of a circle is given by the formula: $A = \pi r^2$